

Lie algebras and symmetric tensor categories

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The study of finite groups evolves from the study of the structure by themselves to the development of their representation theory. The family of representations of a group over a fixed field has several properties: it is closed by finite direct sums, by tensor products, it contains the dual space and the tensor product of two representations is (naturally) isomorphic to the tensor product of the same representations in the opposite order. These properties are abstracted to give the notion of a symmetric tensor category. When the characteristic of the field does not divide the order of the group, the category of representations is semisimple: every representation can be written as a direct sum of simple representations. The smallest example where this does not happen is the category of representations of a cyclic group of prime order over a field whose characteristic is exactly that prime. Through a process of semisimplification we obtain a quotient category which is semisimple, and still tensor symmetric, nowadays called the Verlinde category. Being symmetric, objects from Lie theory can be considered, which give rise to new tensor categories via their representations. We are particularly interested in Lie algebras. Due to a celebrated result by Deligne, symmetric tensor categories of moderate growth over (algebraically closed) fields of characteristic zero correspond to categories of representations of affine algebraic supergroups. Once we move to positive characteristic, we need to take into account the Verlinde category: Coulembier-Etingof-Ostrik proved recently that every such symmetric tensor category is the one of representations of an algebraic group in the Verlinde category, under some restrictions. Thus, we wonder how to describe algebraic groups, which in turn leads to the question of how to obtain Lie algebras in this category. The aim of this talk is to present different ways to construct Lie algebras in symmetric tensor categories, focused on the case of the Verlinde category.

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