

The Aldous-Lyons Conjecture

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A sequence of finite graphs (G_n) is said to Benjamini-Schramm converge to a random rooted graph (G, o) if the radius r neighborhood of a uniformly random vertex v in G_n , converges in distribution to the radius r neighborhood of the root o in G , for every r . In this case, the random rooted graph (G, o) is said to be sofic. A finitely generated group is sofic if one (and hence all) of its Cayley graphs is sofic. All sofic random rooted graphs satisfy the Mass Transport Principle, that is, they are unimodular. Aldous and Lyons asked whether the converse holds: are all unimodular random rooted graphs sofic? In joint work with Michael Chapman, Alex Lubotzky and Thomas Vidick, we find the answer is no. Our proof is modeled on the solution to Tsirelson's problem in the $\text{MIP}^*=\text{RE}$ paper.

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