

Restriction estimates using decoupling theorems and incidence inequalities for tubes

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Suppose f is a function with Fourier transform supported on the unit sphere in R^d , Elias Stein conjectured in the 1960s that the L^p norm of f is bounded by the L^p norm of its Fourier transform, for $p > 2d/(d-1)$. We propose to study this conjecture using Bourgain-Demeter decoupling theorems and incidences estimates for tubes. In this talk, we will describe a geometric conjecture on the number of incidences for tubes that would imply Stein's restriction conjecture. We prove this geometric conjecture in R^2 and use it to prove a restriction estimate in R^3 for $p > 3 + 1/7$, which implies Wolff's hairbrush Kakeya estimate (i.e. any Kakeya set in R^3 has Hausdorff dimension at least $5/2$). This is joint work with Shukun Wu.

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