Curve graphs and totally geodesic subvarieties of moduli spaces of Riemann surfaces

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Given a surface, the associated curve graph has vertices corresponding to certain isotopy classes of curves on the surface, and edges for disjoint curves. Starting with work of Masur and Minsky in the late 1990s, curve graphs became a central tool for understanding objects in low dimensional topology and geometry. Since then, their influence has reached far beyond what might have been anticipated. Part of the talk will be an expository account of this remarkable story.Much more recently, non-trivial examples of totally geodesic subvarieties of moduli spaces have been discovered, in work of McMullen-Mukamel-Wright and Eskin-McMullen-Mukamel-Wright. Part of the talk will be an expository account of this story and its connections to dynamics.The talk will conclude with new joint work with Francisco Arana-Herrera showing that the geometry of totally geodesic subvarieties can be understood using curve graphs, and that this is closely intertwined with the remarkably rigid structure of these varieties witnessed by the boundary in the Deligne-Mumford compactification.

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