Plenary Session: Realizations of homology classes and projection areas

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I will explore two questions about projections of geometric objects in 4-dimensional spaces:

1. Let A be a convex body in $\mathbb{R}^4$, and let $(p_{12}, p_{13}, p_{14}, p_{23}, p_{24}, p_{34})$ be the areas of the six coordinate projections of A to $\mathbb{R}^2$. Which six numbers arise in this way?

2. Let S be an irreducible surface in $(\mathbb{P}^1)^4$, and let $(p_{12}, p_{13}, p_{14}, p_{23}, p_{24}, p_{34})$ be the degrees of the six coordinate projections from S to $(\mathbb{P}^1)^2$. Which six numbers arise in this way?

The answers to these questions are governed by the Plücker relations for the Grassmannian $\mathrm{Gr}(2, 4)$ over the triangular hyperfield $\mathbb{T}_2$. These results suggest a general conjecture on homology classes of irreducible surfaces in smooth projective varieties. The talk will be accessible to a general mathematical audience. Joint work with Daoji Huang, Mateusz Michalek, Botong Wang, and Shouda Wang.

In Plenary Talks at the Mathematical Congress of the Americas 2025.

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