

Plenary Session: An application of probability theory for groups to 3-dimensional manifolds

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The fundamental group of a 3-dimensional manifold captures information about loops in the manifold. One naturally asks what kind of groups are possible as the fundamental groups of 3-manifolds. For any finite group G , it is well-known that there exists a 3-manifold M with G as a quotient of the fundamental group of M . However, we can ask more detailed questions about the possible finite quotients, e.g. for G and H finite groups, does there exist a 3-manifold group with G as a quotient but not H as a quotient? We answer all such questions. To prove existence of 3-manifolds with certain finite quotients but not others, we use a probabilistic method, by first proving a formula for the distribution of the fundamental group of a random 3-manifold, in the sense of Dunfield-Thurston. This formula comes from our recent work on the moment problem for random groups. This is joint work with Will Sawin.

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