Generalized Gelfand pairs

Silvina Campos

Universidad Nacional de Salta

Let G be a connected Lie group and let K be a compact subgroup of G. It's well known that (G, K) is a Gelfand pair if and only if the regular left representation of G on $L2(G/K)$ is multiplicity free (see [3]). In this case, $L2(G/K)$ is the direct integral of irreducible Hilbert spaces $H\phi$ on the space of spherical functions of positive type. Even more, $H\phi$ is generated by the functions of the form $f * \phi$ where f is a continuous function with compact support. In the last years, different works have put attention on Gelfand pairs of the form $(K \ltimes N, K)$ where N is a nilpotent Lie group and K is a subgroup of the automorphism group $\mathrm{Aut}(N)$ of N. One of the first results, proved in [1], stated that if $(K \ltimes N, K)$ is a Gelfand pair then N is abelian or a 2-step nilpotent group. The notion of the Gelfand pair was generalized for K a non compact subgroup of G. Let $D(G/K)$ be the space of infinitely differentiable functions on G/K with compact support.

Definition 1. (G, K) is a generalized Gelfand pair if any unitary representation (π, H) of G realized in $D'(G/K)$ decomposes as a direct integral of irreducible components without multiplicity.

One of first questions is if the result proved in [1] is true for the non compact case. The answer is negative. In this talk, I introduce a family of generalized Gelfand pairs $(Km \ltimes Nm, Nm)$ where Nm is an $m + 2$-step nilpotent Lie group and Km is isomorphic to the 3-dimensional Heisenberg group. Moreover, I develop the associated spherical analysis computing the set of the spherical distributions, which are eigendistributions of the left invariant differential operators on Nm but unlike the compact case an spherical distribution is not always determined by its set of eigenvalues (see [2]).

MCA 2025 internal code: SS01-T03

References

[1] Benson, C., Jenkins, J., Ratcliff, G. On Gelfand pairs associated with solvable Lie groups, Trans. Amer. Math. Soc. 321 (1990), 85-116.

[2] Campos, S., García, J. and Saal, L. Spherical analysis attached to some m-step nilpotent Lie group, J. Fourier Anal. Appl. 30, Article number: 20 (2024).

[3] Wolf, J. Harmonic analysis on commutative spaces, American Mathematical Sociaty. Mathematical Surveys and Monograohs. 142 (2007).

In Special Session 1 on Applied Harmonic Analysis and Operator Theory at the Mathematical Congress of the Americas 2025.