

SU(2) Yang-Mills-Higgs functional with self-interaction term on 3-manifolds

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I will talk about recent joint work with Daniel Fadel (University of São Paulo) and Luiz Lara (Unicamp), where we study the $SU(2)$ Yang-Mills-Higgs functional with positive coupling constant on 3-manifolds. Motivated by the work of Alessandro Pigati and Daniel Stern (2021) on the $U(1)$ -version of the functional, we also include a scaling parameter. When the 3-manifold is closed and the parameter is small enough, by adapting to our context the min-max method used by Pigati and Stern, we construct non-trivial critical points satisfying energy upper and lower bounds that are natural from the point of view of scaling. Then, over 3-manifolds with bounded geometry, we show that, in the limit as the parameter tends to zero, and under the above-mentioned energy upper bound, a sequence of critical points exhibits concentration phenomenon at a finite collection of points, while the remaining energy goes into an L^2 harmonic 1-form. Moreover, the concentrated energy at each point is accounted for by finitely many “bubbles”, that is, non-trivial critical points on R^3 with the scaling parameter set to 1.

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