

Multiplicity results for the Allen-Cahn equation in compact Riemannian manifolds

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We present results from our recent research on the multiplicity of solutions for the Allen-Cahn equation on compact Riemannian manifolds, with or without boundary, under scalar or vectorial volume constraints. More precisely, we can establish a lower bound on the number of solutions based on certain topological invariants of the manifold, provided that the volume constraint and the temperature parameter are sufficiently small. These results are achieved by combining the Gamma-convergence of the Allen-Cahn functional to the (multi) perimeter functional, classical Lusternik-Schnirelmann and Morse theories, and the concentration property of the "almost minimizers" of the perimeter functional.

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