Interior Regularity of Area-Minimizing Currents within $C^{2,\alpha}$ Submanifolds of the Euclidean space

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We investigate the interior regularity of area-minimizing integral m-currents constrained to embedded $C^{2,\alpha}$ submanifolds $\Sigma \subset \mathbb{R}^{m+n}$ of dimension $m + \bar{n}$. Our principal result establishes that the Hausdorff dimension of the interior singular set of such a current T does not exceed $m - 2$. This finding extends Almgren's partial regularity theorem to settings where the ambient space possesses only $C^{2,\alpha}$ regularity, thereby broadening the scope of regularity theory in geometric measure theory. The proof adapts and refines techniques from the works of De Lellis and Spadaro. A crucial innovation is the construction of a $C^{3,\alpha}$ external center manifold and the development of external normal approximations on it with suitable $C^{2,\alpha}$ estimates. These tools facilitate the application of a frequency function and blow-up analysis, even in the absence of higher regularity in the ambient manifold, via a classical smoothing technique whose employment is possible thanks to the $C^{2,\alpha}$ a priori estimates we already established in the external normal approximation. Additionally, we discuss the implications of our results for Riemannian manifolds via Nash's isometric embedding theorem, highlighting the challenges and prospects of pursuing an intrinsic approach to regularity in lower regularity settings. This is a joint work with Reinaldo Resende.

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