

# On a new absolute version of Siegel's lemma

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The classical Siegel's lemma (1929) asserts the existence of a nontrivial integer solution to an underdetermined integer homogeneous linear system, whose "size" is small as compared to the size of the coefficients of the system. Far-reaching generalizations of this theorem, producing a full basis for the solution space, were obtained over number fields by Bombieri & Vaaler (1983), and over the field of algebraic numbers by Roy & Thunder (1996), where the "size" was measured by a height function. We obtain a new version of Siegel's lemma, bridging the Bombieri & Vaaler and Roy & Thunder results in two ways: (1) our basis lies over a fixed number field as in Bombieri & Vaaler's theorem; (2) our height-bound does not depend on the number field in question as in Roy & Thunder's theorem. Our result does not imply the previously established ones and is not implied by them, and our basis has some additional interesting properties. Our method is quite different from the previous ones, using mainly linear algebra. Joint work with Max Forst.

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