Algebraic geometric LDC codes and non special divisors

María Chara

Universidad Nacional del Litoral

For a linear $[n, k, d]$ code $\mathcal{C}$ over a finite field $\mathbb{F}_q$, the dual code is

$$\mathcal{C}^\perp := \{z \in \mathbb{F}_q^n : z \cdot c = 0 \text{ for all } c \in \mathcal{C}\};$$

where the product is the usual (or Euclidean) dot product. A linear code is said to be *linear complementary dual* (LCD) if the intersection with its dual is the trivial code, that is,

$$\mathcal{C} \cap \mathcal{C}^\perp = \{0\}.$$

In the case of an algebraic geometric (AG) code over a function field F,

$$\mathcal{C} = \mathcal{C}(D, G) = \{(f(P_1), \dots, f(P_n)) \mid f \in \mathcal{L}(G)\}$$

where $D = P_1 + P_2 + \cdots + P_n$ is the sum of pairwise distinct rational places $P_1, P_2, \dots, P_n$, and G is a divisor of degree $2g - 2 < \deg G < n$ with support disjointfrom D, the dual code can be represented as

$$\mathcal{C}^\perp = \mathcal{C}(D, D - G + (\eta))$$

where (η) is an appropriate differential. In this talk we will use a general construction of LCD AG codes, that heavily relies on the existence of non special divisors of degree $g-1$ in a function field, but by making the supports of the $gcd(G, H)$, where $H = D-G+(\eta)$, as small as possible. In fact, we are going to show that allowing places of larger degree on the support of G and H it is possible to construct non special divisors of degree $g-1$ with support in two points for certain function fields. This is a part of a joint work with P. Chiapparoli (Argentina), B. Gatti (Italy), M. Montanucci (Denmark), L.

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