

Session 7 (Symbolic Dynamics and Combinatorial Algebras), Title: "Full Groups of Cantor Minimal Systems: Characters and Invariant Measures"

Constantine Medynets

United States Naval Academy

Given a Cantor minimal dynamical system (X, T) , the topological full group $[[T]]$ consists of all homeomorphisms of X that locally act as powers of T . These groups can be viewed as generalized symmetric groups on the continuous orbit equivalence relation of (X, T) . A series of works by Giordano-Putnam-Skau, Matui, Medynets, Nekrashevych, and others have demonstrated that the algebraic structure of topological full groups completely determines the orbit structure of the underlying systems. This naturally leads to the question of whether the structure of invariant measures, an invariant of orbit equivalence, is similarly reflected in the full group's algebraic properties. In this talk, we present joint work with Artem Dudko (IMPAN) on the classification of characters of topological full groups of Cantor minimal systems. We establish that every extreme character of the commutator subgroup of $[[T]]$ is of the form $\mu(\text{Fix}(g))$, where μ is an ergodic product measure on X^n , thereby confirming Vershik's conjecture for the class of full groups. As a consequence, we show that prime indecomposable characters are in one-to-one correspondence with ergodic measures.

In Special Session 7 on Symbolic Dynamics and Combinatorial Algebras at the Mathematical Congress of the Americas 2025.