

A Generalization of the Poincaré-Hopf Theorem for Filippov Vector Fields on 2-Dimensional Compact Manifolds

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In this talk, we'll explore a significant extension of the Poincaré-Hopf Theorem to Filippov vector fields. We demonstrate that the Poincaré-Hopf Theorem indeed extends to Filippov vector fields defined on compact 2-dimensional manifolds with smooth switching manifolds. To obtain this extension, we introduce a new index definition that includes singularities unique to Filippov vector fields, such as pseudo-equilibria and tangential singularities. This new index extends the classical definition for singularities in smooth vector fields, allowing it to include Filippov singularities with smooth switching manifolds. The construction of this index relies on an invariance property under a regularization process, which enables us to establish all classical index properties. As a result, we obtain a variant of the Hairy Ball Theorem: "any Filippov vector field on a sphere with a smooth switching manifold must have at least one singularity (in the Filippov sense) with a positive index".

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