

Generalizations of the Implicit and Inverse Function Theorems: A Topological and Homological Approach

Luiz Roberto Hartmann Junior

Universidade Federal de São Carlos

The Implicit and Inverse Function Theorems are cornerstones of mathematical analysis, providing essential tools for understanding the local behavior of functions and solving equations. While classical versions of these theorems require continuous differentiability, recent research has explored their generalizations to settings where functions are merely continuous or differentiable, but not necessarily C^1 . This talk will survey key results from recent works that extend these theorems in novel directions. We begin with a homological approach to the Implicit Function Theorem for continuous maps between topological manifolds, from Biasi, Gutierrez e dos Santos (2008), where assumptions on the degree of the maps replace traditional differentiability conditions. Next, we explore the Inverse Function Theorem for open and discrete continuous maps, showing how local invertibility can be established without differentiability assumptions, this was presented by Barreto, Hartmann and Fenille (2016). We then discuss extensions of these results to differentiable functions, where the lack of continuous differentiability is compensated by conditions on the Jacobian or the degree of the map, presented by (Radulescu and Radulescu (1989); Fenille and Velloso (2021)).

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