Generic graded contractions of Lie algebras

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Contractions and degenerations of Lie algebras have been extensively studied since the 1950s. The idea is to pass from one Lie algebra to another by some kind of limit process. Another type of contractions was introduced by de Montigny and Patera (1991) in the context of Lie algebras over a field $\mathbb{F}$ equipped with a grading by an abelian group G. This type of contractions consists in modifying the Lie bracket of $\mathfrak{L} = \bigoplus_{g \in G} \mathfrak{L}_g$ by a scalar factor $\varepsilon(g,h)$, $g,h \in G$, as follows:

$$[x,y]^{\varepsilon} := \varepsilon(g,h)[x,y] \quad \text{for all} \quad x \in \mathfrak{L}_g, y \in \mathfrak{L}_h.$$

The function $\varepsilon\colon G \times G \to \mathbb{F}$ must satisfy certain conditions to ensure that the new bracket satisfies the antisymmetry and Jacobi identity. We are interested in *generic* graded contractions, i.e., those that produce a new Lie bracket for any G-graded Lie algebra. They can be viewed as lax monoidal structures on the identity endofunctor of the category of G-graded vector spaces. We show that generic graded contractions with a fixed support S are classified by a certain abelian group $\mathrm{H}^2_S(\mathbb{F}^\times)$, which we explicitly describe. We also analyze which generic graded contractions define graded degenerations of a given G-graded Lie algebra. This talk is based on a joint work with S. Koval.

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