

Entropy numbers and box dimension in Banach spaces

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In this talk, we will deal with entropy numbers and box dimensions of sets in Banach spaces, with a focus on the images of balls under specific mappings. We will see first that the box dimension of a connected and relatively compact set is finite if and only if its entropy numbers have exponential decay. Then, we will consider the image of a ball under a homogeneous polynomial, showing that its box dimension is finite if and only if it spans a finite-dimensional subspace. This fact is trivial for linear operators, but turns out to be rather involved for polynomials of degree greater than one. Moreover, it is false for holomorphic functions. Finally, we will present a connection between the entropy numbers of the image of a holomorphic function and those of the polynomials of its Taylor series expansion. This is a joint work with Carlos D'Andrea, Leodan A. Torres and Pablo Turco.

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