On the fundamental functions of general Dirichlet series bases

Silvia Lassalle

Universidad de San Andrés

In this talk, we explore the behavior of the fundamental functions associated with the canonical basis $\mathcal{B} = e^{-\lambda_n s}$ in the different Hardy spaces $\mathcal{H}_p^\lambda$ of general Dirichlet series, determined by a frequency sequence $\lambda = (\lambda_n)$. These functions, namely the lower/upper democracy and super-democracy functions, play a crucial role in understanding various basis properties.For the classical Hardy spaces $\mathcal{H}_p$ of ordinary Dirichlet series, where the canonical basis takes the form $\mathcal{B} = n^{-s}$, we provide the precise asymptotic behavior of all fundamental functions. The obtained estimates reveal that the canonical basis exhibits "good properties" in $\mathcal{H}_p$ only for $p = 2$, i.e., when the basis is orthonormal.For the general case $\mathcal{B} = e^{-\lambda_n s}$, as expected, the asymptotic behavior of the fundamental functions depends on the chosen frequency. In this setting, we establish sharp lower and upper bounds for all possible behaviors.As a byproduct, we use these estimates to show that, for a wide class of frequencies λ, the basis $\mathcal{B}$ does not possess any of the most commonly studied greedy properties, except when restricted to the Hilbert space $\mathcal{H}_2^\lambda$.This is a joint work with Daniel Carando and Leandro Milne.

In Special Session 10 on Nonlinear Analysis on Banach Spaces at the Mathematical Congress of the Americas 2025.

MCA 2025 internal code: SS10-T06