Transportation Cost Spaces and Stochastic Trees

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Given a finite metric space M one can define the corresponding transportation cost space $\mathcal{F}(M)$ as the normed linear space of transportation problems on M. Roughly speaking, a transportation problem can be understood as a supply/demand configuration on M where the norm of the transportation problem is the lowest cost of transporting goods from locations with a surplus to those with shortages. In this setting, an important line of research is studying the relation between transportation cost spaces and ℓ_1. A core problem posed by S. Dilworth, D. Kutzarova, and M. Ostrovskii is finding a condition on a metric space M equivalent to $\mathcal{F}(M)$ being Banach-Mazur close to ℓ_1^N in the corresponding dimension. In this talk, we discuss our recent work where a partial solution to this problem is obtained by examining tree-like structure within the underlying metric space. Tangential to this result, we have also developed a new technique that, potentially, could serve as a step toward a complete solution to the problem of Dilworth, Kutzarova, and Ostrovskii. We conclude by discussing two applications of this technique: finding an asymptotically tight upper bound of the ℓ_1^N-distortion of the Laakso graphs, and proving that finite hyperbolic approximations of doubling metric spaces have uniformly bounded ℓ_1^N-distortion. This is joint work with Ruben Medina.

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