

Coherent Pairs of Measures of the Second Kind on the Unit Circle: New Examples, Sobolev Orthogonality, and Recent Developments

Lino G. Garza

Universidad de Monterrey

This work synthesizes the development of the theory of coherent pairs of measures (μ_0, μ_1) of the second kind supported on the unit circle, tracing its evolution from foundational results to recent contributions. A review of classical works is presented, followed by recent advances in recurrence relations, Sobolev orthogonality, and connection coefficients. Special attention is given to the classification of such pairs, the role of algebraic and differential techniques, and the construction of explicit examples. Open problems are discussed.

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