

Jacobi-Sobolev Polynomials and Electrostatic Interpretation

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We explore a sequence of monic polynomials orthogonal with respect to a Sobolev-type inner product that combines a Jacobi weight function and discrete derivatives evaluated at specific points outside the interval of orthogonality. A connection formula linking these Sobolev polynomials to the classical Jacobi polynomials is established. Additionally, we derive ladder operators and a second-order differential equation with polynomial coefficients satisfied by the Sobolev polynomials. Conditions are provided under which the zeros of these polynomials correspond to the equilibrium configuration of unit charges in a logarithmic electrostatic potential. Examples illustrating this interpretation are included.

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