Sequentially-ordered Sobolev inner product and Laguerre-Sobolev polynomials

Juan Hernández

Universidad Autónoma de Santo Domingo

We study the sequence of polynomials $\{S_n\}_{n\geq 0}$ orthogonal with respect to the general discrete Sobolev-type inner product

$$\langle f,g\rangle_{\mathsf{s}}=\int f(x)g(x)d\mu(x)+\sum_{j=1}^{N}\sum_{k=0}^{d_j}\lambda_{j,k}f^{(k)}(c_j)g^{(k)}(c_j),$$

where μ is a finite Borel measure whose support $\operatorname{supp}(\mu)$ is an infinite set of the real line, $\lambda_{j,k}\geq 0$, and the mass points c_i, $i=1,\ldots,N$ are real values outside the interior of the convex hull of $\operatorname{supp}(\mu)$ ($c_i\in RR\setminus(\mathbf{C}_h(\operatorname{supp}(\mu)))^{\circ}$). Under some restriction of order in the discrete part of $\langle\cdot,\cdot\rangle_{\mathsf{s}}$, we prove that S_n has at least $n-d^*$ zeros on $(\mathbf{C}_h(\operatorname{supp}(\mu)))^{\circ}$, being d^* the number of termsb in the discrete part of $\langle\cdot,\cdot\rangle_{\mathsf{s}}$. Finally, we obtain the outer relative asymptotic for $\{S_n\}$ in the case the measure μ is the classical Laguerre measure and for each mass point only one order derivative appear in the discrete part of $\langle\cdot,\cdot\rangle_{\mathsf{s}}$.

In Special Session 11 on Non-Standard Orthogonal Polynomials, Special Functions, and Harmonic Analysis: Recent Trends and Applications at the Mathematical Congress of the Americas 2025.

MCA 2025 internal code: SS11-T08