Multi-tilings and the geometry of numbers, via the Siegel-Bombieri approach

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Abstract. We extend the Bombieri-Siegel formula from the geometry of numbers, by studying a lattice sum of the cross-covariogram of any two bounded measurable sets $A, B \subset R^d$. Using a variation of Poisson summation, our extension also refines the summation index of the Bombieri-Siegel formula, allowing us to obtain some interesting applications. One of the consequences of these results is a new characterization of multi-tiling Euclidean space by translations of a compact set. Another consequence is a spectral formula for the volume of a compact set. Finally, we give an application to additive combinatorics, namely an identity for finite sums of discrete covariograms over any set of integer points in the integer lattice. As a consequence, we arrive at an equivalent condition for multi-tiling the integer lattice by any given finite set of integer points. This is joint work with Michel Faleiros Martins. `https://sites.google.com/site/sinairobins`

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