

On the homothety conjecture for the body of flotation and the body of buoyancy on a plane

Bartłomiej Zawalski

Kent State University

We investigate several closely related "homothety conjectures" for convex bodies on a plane. Using the modern language of differential geometry, we systematically derive the fundamental properties of bodies of flotation, bodies of buoyancy, and bodies of illumination. As a direct consequence of our results, we show that if the body of flotation is homothetic to the body of buoyancy, and if every chord of flotation cuts off from the boundary exactly $\frac{1}{3}$ of its total affine arc length, then K is an ellipse. We also provide natural affine counterparts of the classical theorems on the floating body problem from the Scottish Book due to H. Auerbach. In particular, we obtain an affine counterpart of Zindler carousels introduced by J. Bracho, L. Montejano, and D. Oliveros.

In Special Session 12 on Harmonic Analysis in Geometric Tomography at the Mathematical Congress of the Americas 2025.