

No-dimensional Helly theorem

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The classical Helly theorem states that if the intersection of any subfamily of a finite family of convex sets in $\mathbb{R}^d$, consisting of at most $d + 1$ sets, contains a common point, then all the sets in the family have at least one common point. We will discuss a dimension-free version of this celebrated result in Banach spaces, which essentially states that if the intersections of all subfamilies of a fixed size, say k , contain a point in a bounded region, say the unit ball, then the ε_k -neighborhoods of all the sets have a common point. We will show that ε_k forms a null sequence in certain Banach spaces, e.g., uniformly convex ones.

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