

## Simplicial Resolutions of the Quadratic Power of Monomial Ideals

Susan Cooper

University of Manitoba

Diana Taylor established a technique to construct a free resolution of an ideal  $I$  generated by  $q$  monomials using the simplicial chain maps of a simplex on  $q$  vertices. Work of Bayer, Peeva and Sturmfels later extended Taylor's work to show that as long as such a simplicial complex satisfies certain homological conditions, it can support a free resolution of  $I$ . The idea of using the structure of  $I$  to further find data about the Betti numbers of powers of  $I$  becomes quite delicate quickly. Cooper, El Khoury, Faridi, Mayes-Tang, Morey, Sega and Spiroff constructed a complex labeled with the generators of  $I^r$  which supports a free resolution of  $I^r$  when the ideal  $I$  is square-free. In this talk, we generalize these efforts for the square of any monomial ideal. As a result, we obtain improved bounds on the projective dimension and the Betti numbers of  $I^2$ . Introducing permutation ideals allows us to discuss extremal conditions. This is joint work with S. Faridi and H. Mahmood.

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