

Scarf complex of powers of extremal ideals

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We prove that a minimal free resolution of $\mathcal{E}_q^3$, the 3rd power of the extremal ideal $\mathcal{E}_q$, on q generators, is supported by $\mathbb{S}_q^3 = \text{Scarf}(\mathcal{E}_q^3)$, the Scarf complex of $\mathcal{E}_q^3$. Thus, the f -vector of $\mathbb{S}_q^3$ equals the betti numbers of $\mathcal{E}_q^3$. Since $\mathcal{E}_q^r$ has the maximum betti numbers among the r -th power of any square-free monomial ideal with q generators, this f -vector gives us a sharp upper bound on the betti numbers for third powers, which is an exponential improvement over previously known bounds. We provide a complete description of $\mathbb{S}_q^3$, entirely in terms of discrete geometry, and a partial description of $\mathbb{S}_q^r = \text{Scarf}(\mathcal{E}_q^r)$.

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