

Edge ideals with linear quotients without homological linear quotients

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Given a monomial ideal I . The k -th homological shift ideal $HS_k(I)$ of I is generated by the multigrading shifts at homological degree k in the minimal free resolution of I . In particular, $HS_0(I) = I$. The ideal I is said to have homological linear quotients if $HS_k(I)$ has linear quotients for any $k \geq 0$. It is of interest to find all graphs G whose edge ideals have homological linear quotients. By the classic Fröberg's theorem, such a G must be co-chordal. Some known such graphs are the complement of forests, complement of proper interval graphs (Ficarra, Herzog '23), and complement of block graphs (Taghipour, Bayati, Rahmati '24). We construct a family of co-chordal graphs $\{H_n\}_{n \geq 6}$ and prove that if the edge ideal $I(G)$ has homological linear quotients, then G is co-chordal and H_n -free for any $n \geq 6$. We conjecture that the converse also holds. Moreover, we analyze the patterns of pairs (G, k) such that $HS_k(I(G))$ has linear quotients. This is joint work with Kanoy Kumar Das and Aryaman Maithani.

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