

13 - Sparse systems with high local multiplicity

Alicia Dickenstein

Universidad de Buenos Aires

Consider a sparse system of n Laurent polynomials in n variables with complex coefficients and support in a finite lattice set A . The maximal number of isolated roots of the system in the complex n -torus is known to be the normalized volume of the convex hull of A (the BKK bound). In joint work with Frédéric Bihan and Jens Forsgård, we explore the following question: if the cardinality of A equals $n+m+1$, what is the maximum local intersection multiplicity at one point in the torus in terms of n and m ? This study was initiated by Gabrielov in the multivariate case. We give an upper bound that is always sharp when $m=1$ and, under a generic technical hypothesis, it is considerably smaller for any dimension n and codimension m . We also present, for any value of n and m , a particular sparse system with high local multiplicity with exponents in the vertices of a cyclic polytope and we explain the rationale of our choice. Our work raises several interesting questions.

In Special Session 13 on Influences of Combinatorics and Topology in Commutative Algebra at the Mathematical Congress of the Americas 2025.