

Simplicial complexes with many facets are vertex decomposable

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Suppose X is a pure simplicial complex on n vertices having dimension d and codimension $c = n - d - 1$. We show that if X has at least $\binom{n}{c}$ facets, then X is vertex decomposable. This improves a result of Terai and Yoshida, who used algebraic methods to prove that these conditions imply that X is Cohen-Macaulay. Our proof uses elementary combinatorial and topological tools. We explore applications to Stanley-Reisner rings and Simon's Conjecture, and discuss connections to other results from the literature. This is joint work with Ritika Nair, Jay Schweig, Adam Van Tuyl, and Russ Woodroofe.

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