

Regular Sequences and Hilbert Series of Edge Ideals

Susan Morey

Texas State University

Given a square-free monomial ideal I in a polynomial ring R , finding elements that are regular on R/I is relatively straightforward, but finding regular sequences is significantly more difficult. The focus of this talk will be identifying regular sequences of linear forms and using such sequences to gain information about the Hilbert Series of R/I . There is a well-known formula connecting the h-vector formed by the coefficients of the Hilbert Series of R/I to an f-vector, which counts faces of the Stanley-Reisner complex associated to the ideal, through an alternating sum with binomial coefficients. Using the regular sequences provides conditions under which the Hilbert series can be more directly computed from the f-vector corresponding to a smaller graph. When R/I is Cohen-Macaulay and there is a maximum regular sequence satisfying given conditions, then the h-vector is precisely the f-vector corresponding to a smaller graph. This allows one to identify what the coefficients of the h-vector “count” in the Cohen-Macaulay case, where the coefficients are known to be non-negative.

In Special Session 13 on Influences of Combinatorics and Topology in Commutative Algebra at the Mathematical Congress of the Americas 2025.