Grothendieck Shenanigans: Permutons from pipe dreams via integrable probability

Alejandro Morales

Université du Québec à Montréal

We study random permutations arising from reduced pipe dreams. Our main model is motivated by Grothendieck polynomials with parameter beta=1 arising in K-theory of the flag variety. The probability weight of a permutation is proportional to the principal specialization (setting all variables to 1) of the corresponding Grothendieck polynomial. By mapping this random permutation to a version of TASEP (Totally Asymmetric Simple Exclusion Process), we describe the limiting permuton and fluctuations around it as the order n of the permutation grows to infinity. The fluctuations are of order $n^(1/3)$ and have the Tracy-Widom GUE distribution, which places this algebraic (K-theoretic) model into the Kardar-Parisi-Zhang universality class.We also investigate non-reduced pipe dreams and make progress on a recent open problem on the asymptotic number of inversions of the resulting permutation. Inspired by Stanley's question for the maximal value of principal specializations of Schubert polynomials, we resolve the analogous question for beta=1 Grothendieck polynomials, and provide bounds for general beta.This is joint work with Greta Panova, LeoPetrov, and Damir Yeliussizov.

In Special Session 15 on Integrable Probability and KPZ Universality at the Mathematical Congress of the Americas 2025.

MCA 2025 internal code: SS15-T09