Invariant subspaces of nilpotent operators

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Our interest is in the chain of categories $\mathcal{S}(n)$ where n is a natural number. The objects are pairs $X = (U, V)$, where V is a finite-dimensional vector space with a nilpotent operator T with $T^n = 0$, and U is a subspace of V such that $T(U) \subset U$. To an object $X = (U, V)$ we assign three numbers, $uX = \dim U$ (for the subspace), $wX = \dim V/U$ (for the factor) and $bX = \dim \mathrm{Ker}T$ (for the operator). Actually, instead of looking at the reference space $\mathbb{R}^3$ with the triples (uX, wX, bX), we will focus the attention to the corresponding projective space $\mathbb{T}(n)$ which contains for a non-zero object X the level-colevel pair $(uX/bX, wX/bX)$ supporting the object X.We use $\mathbb{T}(n)$ to visualize part of the categorical structure of $\mathcal{S}(n)$: Each $\mathbb{T}(n)$ is pictured as an equilateral triangle since the duality D gives rise to a reflection symmetry, while the rotation on $\mathbb{T}(n)$ by 120° degrees highlights the operation on the objects given by the square τ_n^2 of the Auslander-Reiten translation.Consider the triangles $\mathbb{T}(n)$ as n increases: For $n < 6$, the categories $\mathcal{S}(n)$ have finite type, so there are only finitely many points in $\mathbb{T}(n)$. But in the wild cases where $n > 6$, there are large regions in $\mathbb{T}(n)$ which are densely populated by indecomposables, although a lot is still unknown. The borderline case $\mathcal{S}(6)$ is tame of tubular type $\mathbb{E}_8$. Here, surprisingly for us, all indecomposable objects lie on 12 central lines in $\mathbb{T}(6)$.This talk is about a joint paper with Claus Michael Ringel, see arxiv.org/abs/2405.18592.

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