Maximal almost rigid modules over gentle algebras

Emily Gunawan

University of Massachusetts Lowell

We study maximal almost rigid modules over a gentle algebra A. We prove that the number of indecomposable direct summands of every maximal almost rigid A-module is equal to the sum of the number of vertices and the number of arrows of the Gabriel quiver of A. Moreover, the algebra A, considered as an A-module, can be completed to a maximal almost rigid module in a unique way. Gentle algebras are precisely the tiling algebras of surfaces with marked points. We show that the (permissible) triangulations of the surface of A are in bijection with the maximal almost rigid A-modules. Furthermore, we study the endomorphism algebra $C = End_A T$ of a maximal almost rigid module T. We construct a fully faithful functor G: mod A to mod $\overline{A}$ into the module category of a bigger gentle algebra $\overline{A}$ and show that G maps maximal almost rigid A-modules to tilting $\overline{A}$-modules. In particular, C and $\overline{A}$ are derived equivalent and C is gentle. After giving a geometric realization of the functor G, we obtain a tiling $G(\mathcal{T})$ of the surface of $\overline{A}$ as the image of the triangulation $\mathcal{T}$ corresponding to T. We then show that the tiling algebra of $G(\mathcal{T})$ is C. Moreover, the tiling algebra of $\mathcal{T}$ is obtained algebraically from C as the tensor algebra with respect to the C-bimodule $Ext^2_C(DC, C)$, which also is fundamental in cluster-tilting theory.

In Special Session 16 on Special Session in Representation Theory of Algebras at the Mathematical Congress of the Americas 2025.

MCA 2025 internal code: SS16-T05