

Exact weights and path metrics for triangulated categories and the derived category of persistence modules

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This is a joint work with P. Bubenik at the University of Florida. We define exact weights on a triangulated category to be nonnegative functions on objects satisfying a subadditivity condition with respect to exact triangles. Such weights induce a metric on objects in the triangulated category, which we call a path metric. Our exact weights generalize the rank functions of J. Chuang and A. Lazarev and are analogous to the exact weights for an exact category given by P. Bubenik and J. Scott and D. Stanley. We show that cohomological functors from a triangulated category to an abelian category with an additive weight induce an exact weight on the triangulated category. We prove that triangle equivalences induce an isometry for the path metrics induced by cohomological functors. In the perfectly generated or compactly generated case, we use Brown representability to express the exact weight on the triangulated category. We give three characterizations of exactness for a weight on a triangulated category and show that they are equivalent. We also define Wasserstein distances for triangulated categories. Finally, we apply our work to derived categories of persistence modules and to representations of continuous quivers of type A.

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