On the boundary polynomial of a graph

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In this work, we introduce the boundary polynomial of a graph G as the ordinary generating function in two variables $B(G;x,y) := \sum_{S\subseteq V(G)} x^{|B(S)|}y^{|S|}$, where $B(S)$ denotes the outer boundary of S. We investigate this graph polynomial obtaining some algebraic properties of the polynomial as well as finding some graph parameters encoded in the polynomial. We study its coefficients and show some numerical evaluations related to some graph parameters. In particular, we show that the boundary polynomial algebraically encloses some combinatorial parameter of the graph like domination number, Roman domination number, vertex connectivity and differential of the graph. Furthermore, we obtain some algebraic relation between some binary operation of two graphs and its factor graphs that allow us to compute the boundary polynomial for some classic families of graphs. We also establish some relationships between $B(G;x,y)$ and $B(G';x,y)$ for the graphs G' obtained by removing, adding, and subdividing an edge from G. Besides, we prove that a graph G has an isolated vertex if and only if its boundary polynomial has a factor $(y+1)$. Finally, we show that the classes of complete, complete without one edge, empty, path, cycle, wheel, star, and double-star graphs are characterized by the boundary polynomial. As a final comment, i would like to pointed out that the boundary polynomial provides an algebraic way to algebraically solve the Vizing's conjecture whenever someone obtains a combinatorics/algebraic relation between the boundary polynomial of the Cartesian product of two graphs in term of its factors

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