

The Saturation Spectrum of Berge Stars

Daniel Johnston

Trinity College

Given two k -uniform hypergraphs F and H , we say H is F -saturated if H does not contain a copy of F , but $H + e$ does for any edge $e \in E(\overline{H})$. Given a graph G and a hypergraph H embedded on the same vertex set, we say H is a Berge- G if there exists a bijection $\phi : E(G) \rightarrow E(H)$ such that $e \subseteq \phi(e)$ for all $e \in E(G)$.

In this talk, we look at the saturation spectrum for 3-uniform Berge stars. The n -vertex saturation spectrum of a hypergraph F is the set of all integers m such that there exists an n -vertex m -edge F -saturated hypergraph. We completely determine the saturation spectrum for 3-uniform Berge- $K_{1,\ell}$ for $1 \leq \ell \leq 4$, and for $\ell = 5$ when $5 \mid n$. We also determine all but a constant number of values in the spectrum for 3-uniform Berge- $K_{1,\ell}$ for all $\ell \geq 5$.

In Special Session 17 on Graph Theory and its Applications at the Mathematical Congress of the Americas 2025.