Embedding theorems as a bridge between supertraces and supergeometry

Claudemir Fideles Bezerra Júnior

IMECC-UNICAMP

There are a number of important theorems on the subject of matrix embeddings. Many of these results want to answer the following question: When does a given ring have an embedding into $n \times n$ matrices over some commutative ring? An obvious necessary condition is that the ring must satisfy the polynomial identities of $n \times n$ matrices, but such condition is not sufficient for $n > 2$. Procesi proved that an algebra R with trace can be embedded into $n \times n$ matrices over some commutative ring if and only if it satisfies the Cayley-Hamilton identity of degree n. Here we recall that the Cayley–Hamilton polynomial of a matrix a can be written as a polynomial whose coefficients are polynomials in the traces of a^k, $k \geq 1$. Such algebras are called Cayely-Hamilton algebras, and interestingly enough the image of this embedding coincides with a ring of invariants, suggesting some geometrical applications. Over a field of characteristic 0, let E denote the infinite-dimensional Grassmann algebra. Consider a power-associative, finite-dimensional, $\mathbb{Z}_2$-graded central simple algebra A, and a supertrace algebra B belonging to the same variety as $A \otimes E$. In this talk, we will present conditions on B that ensure it can be embedded into $A \otimes \Xi$, where Ξ is a supercommutative algebra referred to as the A-universal supermap of B. This embedding is established provided that B satisfies all the supertrace identities of $A \otimes E$. Finally, we will discuss how this result can be applied to certain problems in geometry. The talk is based on a joint paper with Charles Almeida and Lucio Centrone, and was supported by FAPESP grant No. 2023/04011-9.

In Special Session 18 on Automorphisms, Derivations, and Identities of Algebras at the Mathematical Congress of the Americas 2025.

MCA 2025 internal code: SS18-T03