A Criterion for the algebraic density property of affine SL(2,C)-manifolds

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Let B be an affine integral domain over $\mathbb{C}$. A *fundamental pair* for B is a pair (D, U) of locally nilpotent derivations such that D, U and $E = [D, U]$ form an $\mathfrak{sl}_2$-triple. We describe the structure of B induced by a fundamental pair and show that E must be semisimple. Consequently, any fundamental pair induces an algebraic SL_2-action on $X = \mathrm{Spec}(B)$. We prove an algebraic criterion characterizing under which conditions the fundamental pair (D, U), respectively, the triple (D, U, E), is *compatible* in a technical sense that allows us to construct many vector fields on X. In case X is smooth, X is said to have the *algebraic density property* if the Lie algebra generated by the complete polynomial vector fields on X coincides with the Lie algebra of all polynomial vector fields on X. This definition is due to Varolin. It can be difficult to determine whether a given variety has the algebraic density property. Our criterion enables us to prove the algebraic density property for certain widely studied classes of SL_2-varieties, including the Classical Calogero–Moser spaces arising in physics.

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