

Generalization of the Kraft-Russell Generic Equivalence Theorem

Daniel Daigle

University of Ottawa

The Kraft-Russell Generic Equivalence Theorem states that two affine morphisms of varieties $X \rightarrow S$ and $Y \rightarrow S$ with isomorphic closed fibers become isomorphic after a dominant étale base change $S' \rightarrow S$. Kaliman (2020) generalized this result by weakening some of the hypotheses. We further extend the result in several ways: Instead of considering the existence of isomorphisms (between closed fibers or globally), we consider the existence of morphisms of schemes having a given property P . Here, P is a property of morphisms of schemes that satisfies certain stability conditions. We show that there is a large set of properties P for which the theorem is valid (the property of being an isomorphism is an example). Instead of considering two schemes over S (such as $X \rightarrow S$ and $Y \rightarrow S$ in the above statement), we establish the result for arbitrary commutative diagrams of schemes over S . We offer an equivariant version of the result, where schemes are equipped with actions by an algebraic structure A and morphisms are required to be A -equivariant. The structure A can be any algebraic object defined by finitely many commutative diagrams. (For instance, A may be a group scheme or an algebraic group.)

In Special Session 18 on Automorphisms, Derivations, and Identities of Algebras at the Mathematical Congress of the Americas 2025.