

Automorphisms of the category of free finitely generated algebras

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Let Θ be an arbitrary variety of algebras and Θ^0 the category of all free finitely generated algebras in Θ . The group $Aut(\Theta^0)$ of automorphisms of the category Θ^0 plays an important role in universal algebraic geometry. We refer to [2, 3] for relevant notions in universal algebraic geometry. It turns out that for a wide class of varieties, the group $Aut(\Theta^0)$ can be decomposed into a product of the normal subgroup $Inn(\Theta^0)$ of inner automorphisms and the subgroup $St(\Theta^0)$ of strongly stable automorphisms. The method of verbal operations provides a machinery to calculate the group $St(\Theta^0)$ of strongly stable automorphism (see [4, 5, 6]). This allows us to find out the structure of the group of outer automorphisms $Out(\Theta^0) = Aut(\Theta^0)/Inn(\Theta^0)$, which, in some sense, measures the difference between two types of geometrical equivalences of algebras (see [3]). In this talk we give some clarifying remarks describing the place of Θ^0 and $Aut(\Theta^0)$ in the general set up of the universal algebraic geometry and discuss new results concerning the group of strongly stable automorphisms for the variety of non-associative algebras with unit (see [1]).

References

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