

Aproximation and stability results for the parabolic FitzHugh-Nagumo system with rapidly oscillating terms

Eduardo Cerpa

Pontificia Universidad Católica de Chile

The use of high-frequency currents in neurostimulation has received increased attention in recent years due to its varied effects on tissues and cells. Neurons are commonly modeled as nonlinear systems, and questions such as stability can thus be mathematically addressed with methods as the averaging and Lyapunov functions. In this talk, we consider the one-dimensional FitzHugh-Nagumo system under the effects of a source composed of two terms that are sinusoidal in time and quadratically decaying in space. To study this setting we develop a new averaging strategy to prove that, when the frequencies involved are sufficiently high, the full system can be approximated by an explicit highly-oscillatory term plus the solution of a simpler (albeit non-autonomous) system. This decomposition can be seen as a stability result around a varying trajectory. One of the main novelties of the proofs presented here is an extension of the contracting rectangles method to the case of parabolic equations with space and time-depending coefficients. Similar approach can be applied to deal with the same system posed on the half-line with rapidly oscillating Neumann boundary conditions.

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