Ergodic and fixed point theorems for some sequences and mappings in Banach spaces

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Let X be a real Banach space with norm $|\cdot|$. A sequence $\{x_n\}$ in X is called a nonexpansive sequence if
$|x_{i+1} - x_{j+1}| \leq |x_i - x_j|, \qquad \forall i, j \geq 1$. When X is a real Hilbert space H, ergodic theorems for such sequences were proved by Djafari Rouhani in 1981, and it was stated as an open problem whether those results could be extended to Banach spaces. By introducing the notion of ϕ-nonexpansive sequences and mappings in X, we give a partial affirmative answer to this open question. We prove ergodic, weak and strong convergence theorems, as well as fixed point theorems for such sequences and mappings in Banach spaces.

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