Neural Control of Discrete Weak Formulations of PDEs

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We introduce the concept of neural control of discrete weak formulations of Partial Differential Equations (PDEs), in which finite element discretizations are intervened using neural-network weight functions. The weight functions act as control variables that -through the minimization of a cost (or loss) functional– produce discrete solutions incorporating user-defined desirable attributes (e.g., known-data features, remotion of spurious oscillations, or precision at certain quantities of interest). The well-posedness and convergence of the associated constrained optimization problem are analyzed. In particular, we prove under certain conditions that the discrete weak forms are stable and that quasi-minimizing neural controls exist, which converge quasi-optimally. We specialize our analysis in Galerkin, least-squares, and minimal-residual formulations. Elementary numerical experiments support our findings and demonstrate the potential of the framework.

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