

Value Function and Characterization of Controls in Optimal Control Problems with Uncertainty

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We investigate optimal control problems with uncertain parameters, framed within the Riemann-Stieltjes, average, or ensemble optimal control setting. These uncertainties, modeled in a compact metric probability space, appear in the system dynamics, terminal cost, and initial conditions, leading to an infinite-dimensional formulation. By employing a Hilbert space approach, we establish that the value function satisfies the Hamilton-Jacobi-Bellman (HJB) equation as the unique lower semi-continuous proximal solution. Additionally, we extend the Pontryagin Maximum Principle to control-affine systems under uncertainty, characterizing feedback controls and singular arcs. Our results are supported by numerical examples, demonstrating both theoretical and practical significance.

In Special Session 21 on Optimization and Control at the Mathematical Congress of the Americas 2025.