

On measure rigidity of u-Gibbs states

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Physical measures are an important tool in the study of hyperbolic dynamics, governing, for example, the statistical properties of the orbit of almost every point with respect to volume (in the dissipative setting). The well-studied uniformly hyperbolic (Anosov) diffeomorphisms and flows always have ergodic physical measures, whereas the more general class of partially hyperbolic systems lose this property. For these systems, we are instead guaranteed the existence of at least one, and possibly infinite, ergodic u-Gibbs measure(s). In the case of a unique u-Gibbs measure, that measure is automatically physical. Thus, a natural question in the partially hyperbolic setting is the following: under what conditions is there a unique u-Gibbs measure? More generally, which u-Gibbs measures are physical? This question was partially answered in dimension three by Eskin, Potrie, and Zhang. Here we partially extend this result to arbitrary dimensions, and discuss the dichotomy that arises: roughly, a u-Gibbs measure is physical if and only if it is not jointly integrable of some order.

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