

Magnetic trajectories on Heisenberg nilmanifolds

Gabriela Ovando

Universidad Nacional de Rosario

By fixing the usual (left-invariant) metric on the Heisenberg Lie group H_3 , the idea is to provide a comprehensive overview of the behavior of magnetic trajectories for any left-invariant Lorentz force. Thus, we examine closed magnetic trajectories on Heisenberg nilmanifolds of dimension three. We prove that the magnetic curves are solutions of a variational problem globally stated on H_3 . We identify conditions under which a Lagrangian exists. A first step in determining magnetic trajectories through the identity element in H_3 is given by the existence of symmetries of the magnetic equation. It is important to notice that the symmetries can be computed for a magnetic equation on any Riemannian manifold, once the isometry group is known. In fact they constitute the isotropy subgroup of a given action. After that, we determine the closed magnetic trajectories and investigate their existence on compact quotients $\Lambda \backslash H_3$ for any cocompact lattice $\Lambda \subset H_3$. We demonstrate that closed magnetic geodesics are not guaranteed in every homotopy class. Additionally, we provide examples of compact quotients $\Gamma_k \backslash H_3$ with infinitely many closed magnetic trajectories, as well as compact quotients with no closed non-contractible magnetic trajectories for a given Lorentz force. Time permitting, we shall introduce ideas for the study of the corresponding magnetic flows. In this case, integrability in the Liouville sense depends on the energy level and the Lorentz force. This is a joint work with Mauro Subils, Universidad Nacional de Rosario.

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