SU(2) Yang-Mills-Higgs Functional on 3-Manifolds: Energy Concentration and Min-Max Solutions

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I will present recent joint work with Da Rong Cheng (Miami) and Luiz Lara (Unicamp) on the SU(2) Yang-Mills-Higgs functional with arbitrary positive coupling constant over oriented Riemannian 3-manifolds. This gauge-invariant energy is defined on pairs consisting of a connection on an SU(2)-bundle and a section of the adjoint bundle, and its critical points are solutions to a nonlinear second-order elliptic system modulo gauge. Motivated by the work of Pigati and Stern (2021) on the abelian case, we introduce a scaling parameter and study sequences of critical points of the rescaled functional under natural energy bounds. On 3-manifolds with bounded geometry, we show that, as the parameter tends to zero, energy concentrates at a finite set of points, while the remaining energy is accounted for by the formation of an L2 harmonic 1-form. Around each concentration point, a finite collection of "bubbles" arises—non-trivial critical points on R3 for the rescaled functional with parameter equal to one. An energy gap result, together with a no-neck property, ensures that the total concentrated energy at each point is precisely the sum of the energies of finitely many such bubbles. On closed 3-manifolds, we adapt the min-max construction of Pigati and Stern to the SU(2) setting and, for sufficiently small values of the scaling parameter, establish the existence of non-trivial critical points satisfying energy bounds natural from the scaling perspective. When the manifold is a rational homology 3-sphere, we ensure the occurrence of non-trivial bubbling, which in turn implies the existence of non-trivial critical points on R3. If time permits, I will conclude with open questions and potential extensions to higher dimensions.

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