

Gauge Theory on Hyperkahler Manifolds and 3-Sasakian Links

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Manifolds with special holonomy — such as Calabi-Yau, hyperkahler, G2, and Spin(7)-manifolds — are challenging objects to study. In recent years, geometers have proposed various gauge-theoretic PDE (known as “instanton equations”) on such spaces, partly in an effort to define enumerative invariants. While much work has been done in the Calabi-Yau, G2, and Spin(7) settings, the hyperkahler (HK) case has thus far received less attention. In this talk, we discuss a natural class of Yang-Mills connections over hyperkahler $4n$ -manifolds X , known as $\mathrm{Sp}(n)$ -instantons, that generalize ASD instantons. To model their conical singularities, we relate $\mathrm{Sp}(n)$ -instantons over HK cones to contact instantons over their links, and establish some dimensional reductions. We then prove a “Lewis-type theorem” on asymptotically conical (AC) hyperkahler manifolds X to the following effect: If X admits AC $\mathrm{Sp}(n)$ -instantons, then any Hermitian Yang-Mills connection over X that decays sufficiently rapidly is necessarily an $\mathrm{Sp}(n)$ -instanton. This is joint work with Emily Windes (Oregon).

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