

An asymptotic Nahm transform for $\text{Spin}(7)$ instantons on a torus

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Attempts to extend the Nahm transform from 3-dimensional monopoles and 4-dimensional instantons to generalized instantons on manifolds with holonomy G_2 and $\text{Spin}(7)$ face two fundamental difficulties. The first is in proving a vanishing theorem that allows us to endow the index bundle with a canonical connection, and the second is to prove that the transformed connection is itself a generalized instanton. In this talk, I will give examples of generalized $\text{Spin}(7)$ instantons on an 8-torus where the expected vanishing theorem fails. Nonetheless, after twisting by a high power k of an auxiliary holomorphic line bundle, a Kodaira-type vanishing theorem allows for an ‘asymptotic Nahm transform’. We will compute this asymptotic Nahm transform in the $\text{Spin}(7)$ case via a pseudodifferential approximation to the heat kernel and prove that the transformed curvature enjoys a certain amount of holonomy reduction as $k \rightarrow \infty$: it is holomorphic to zeroth order, $\text{Spin}(7)$ to first order, and there exist examples for which the holonomy is not $\text{Spin}(7)$ to second order.

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